Exam T3 - Thermal Physics II, Prof. G. Palasantzas

- Date: 11-12-2025
- Total number of points 100
- 10 points for taking the exam

Problem 1 (20 points)
Consider a system where its energy E behaves like $E = \alpha|x|^m$ with $\alpha > 0$ and m a positive integer (≥ 1). Calculate the average energy <E> as a function of the temperature T and the exponent m.
You could consider the relations: $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$ and $\Gamma(z+1) = z\Gamma(z)$.

Problem 2 (25 points)
A system of N indistinguishable particles with fixed volume V is at thermal equilibrium at temperature T. Each particle has three internal energy levels $E1 = \varepsilon$, $E2 = 2\varepsilon$ and $E3 = 3\varepsilon$, with degeneracies $g(E1) = g(E3) = 1$, $g(E2) = 2$.
(a: 5 points) Write properly the internal partition function Z_{int} of a single particle.
(b: 10 points) Calculate the internal energy U of the system of the N particles
(c: 10 points) Show that the heat capacity $C_V = \partial U/\partial T$ is given by $C_V = 2NK_B[x^2 e^{-x}/(1+e^{-x})^2]$ with $x = \beta\varepsilon$.

Problem 3 (20 points)
Consider the total partition function Z_N of a classical ideal gas of N (>>1) indistinguishable particles within a volume V at temperature T, and Z_1 ($=V/\lambda_{th}^3$) the single–particle partition function due to translational motion with λ_{th} the thermal wavelength.
(a: 5 points) Write the correct form of the partition function Z_N due to translational motion and discuss briefly the regime of its validity.
(b: 5 points) Prove that chemical potential is given by $\mu = k_B T \ln[N/Z_1]$.
(c: 5 points) Is the chemical potential μ positive or negative?. Justify your answer.
(d: 5 points) Prove that the grand potential $\Phi_G = F - \mu N$ is given by $\Phi_G = -PV$.

Problem 4 (10 points)
Assume that the system of the N particles in Problem 2 can be treated as a classical ideal gas with respect to translational motion within the volume V.
(a 5 points) Write properly the total partition function of the system to include both the contribution of internal degrees of freedom and the translational motion.
(b 5 points) Calculate the heat capacity C_{Total} of the system. For the translational motion use the partition function of a classical ideal gas from Problem 3.

Problem 5 (15 points)
(a 5 points) What is the value of chemical potential of a photon gas at thermal equilibrium?
(b 10 points) Using the partition function of a harmonic oscillator $Z_\omega = e^{-\hbar\omega\beta/2}/(1 - e^{-\hbar\omega\beta})$ and the density of states $g(\omega)\, d\omega = [V\omega^2/\pi^2 c^3]\, d\omega$ for a photon gas in volume V at thermal equilibrium, show that the partition function Z of the photon gas is given by

$$\ln Z = -\frac{V}{\pi^2 c^3}\int_0^\infty \omega^2 \ln\left(1 - e^{-\hbar\omega\beta}\right) d\omega$$